FOURIER THEORY ON LIPSCHITZ CURVES

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The aim of this talk is to indicate how the theory of Fourier multipliers in $L_p(\mathbb{R})$ can be adapted when the real line $\mathbb{R}$ is replaced by a Lipschitz curve γ. Details will appear in [6].

(I) Let us start with a resumé of the usual theory concerning $L_p(\mathbb{R})$.

(Ia) The Fourier transform

$$\hat{f}(\xi) = \int_{\mathbb{R}} e^{-ix\xi} f(x)dx$$

defines a mapping

$$L_1(\mathbb{R}) \xrightarrow{\wedge} C_0$$

where C_0 denotes the space of continuous functions on $(-\infty,\infty)$ which tend to zero at $\pm\infty$. We consider the inverse Fourier transform

$$\check{w}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ix\xi} w(\xi)\, d\xi$$

$$L_p(\mathbb{R}) \xleftarrow{\vee} S$$

where S is the Schwartz space of rapidly decreasing functions on $(-\infty,\infty)$. Then

$$(1) \qquad \int_{\mathbb{R}} f(x)\, \check{w}(x)\, dx = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\xi)\, w(-\xi)\, d\xi$$

for all $f \in L_1(\mathbb{R})$ and $w \in S$, so it is consistent with the case $p = 1$ to define

$$L_p(\mathbb{R}) \xrightarrow{\wedge} S'$$

by

$$\langle \hat{f}, w_- \rangle = 2\pi \int_{\mathbb{R}} f(x)\, \check{w}(x)\, dx\,, \quad w \in S\,,$$

for $1 < p \le \infty$, where $w_-(\xi) = w(-\xi)$ and S' is the space of the tempered distributions. We note that

(2) $\{\check{w} \mid w \in S\}$ is dense in $L_p(\mathbb{R})$, $1 \le p < \infty$, and in $C_0(\mathbb{R})$, from which it is immediate that

(3) $L_p(\mathbb{R}) \xrightarrow{\wedge} S'$ is one-one.

Of course, the following also holds: