THE CHARACTERISTIC FUNCTION OF A UNIFORMLY CONTINUOUS SEMIGROUP

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1. INTRODUCTION

Let $T(t)$ be a uniformly continuous one-parameter semigroup of operators on a separable Hilbert space $\mathcal{H}$. Thus, for each $t \geq 0$, $T(t)$ is a bounded operator on $\mathcal{H}$, $T(t_1)T(t_2) = T(t_1+t_2)$ for each $t_1, t_2 \geq 0$, $T(0) = I$, and $\|T(t) - I\| \to 0$ as $t \to 0^+$. Such a semigroup possesses a bounded infinitesimal generator A, defined as the limit (in norm) of $t^{-1}(T(t) - I)$, as $t \to 0^+$. We can then write $T(t) = \exp(At)$. (See, for example, [2], [4], [5], [7], [8], [9].)

As in [2], we define the following bounded operators on $\mathcal{H}$: $G = A + A^*$, $Q = |G|^{1/2}$, and $J = \operatorname{sgn}(-G)$ (this is the operator S in [2]). We have the relations

$$
(1.1) \qquad \begin{aligned} &JQ^2 = -G, \\ &\frac{d}{dt}(T(t)T(t)^*) = T(t)GT(t)^*, \quad \text{and} \\ &\frac{d}{dt}(T(t)^*T(t)) = T(t)^*GT(t) \; . \end{aligned}
$$

A Krein space $\mathcal{G}$ is defined by taking $\mathcal{G}$ to be the space $J\mathcal{H}$, equipped with the indefinite inner product

$$
(1.2) \qquad [x,y] = (Jx,y) \qquad x,\ y \in \mathcal{G}
$$