HANKEL OPERATORS ON THE PALEY-WIENER SPACE IN $\mathbb{R}^d$

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Let $I^d = (-\pi,\pi)^d = \{\xi \in \mathbb{R}^d : -\pi < \xi_j < \pi , i = 1,\dots,d\}$ and let χ_{I^d} denote the characteristic function of I^d. Denote the Fourier transform of g by $F(g) = \hat{g}$ and the inverse Fourier transform of f by $F^{-1}(f) = \check{f}$:

$$\hat{g}(\xi) = \frac{1}{(2\pi)^{d/2}} \int_{\mathbb{R}^d} g(x)e^{-i\xi\cdot x}dx \tag{1}$$

and

$$\check{f}(x) = \frac{1}{(2\pi)^{d/2}} \int_{\mathbb{R}^d} f(\xi)e^{i\xi\cdot x}d\xi . \tag{2}$$

The Paley-Wiener Space on I^d, $PW(D^d)$, is defined to be the image of $L^2(I^d)$ under F^{-1}, i.e.

$$PW(I^d) = \{F^{-1}(\chi_{I^d}f) : f \in L^2(I^d)\} . \tag{3}$$

As is well known, f is in $PW(I^d)$ if and only if it is the restriction to $\mathbb{R}^d$ of an entire function of exponential type at most $(\pi+\epsilon,\dots,\pi+\epsilon)$ in $\mathbb{C}^d$ which satisfies $\|f\|_2 = \left[\int_{\mathbb{R}^d} |f(x)|^2dx\right]^{1/2} < \infty$.

Let P_{I^d} denote the projection defined by $(P_{I^d}g)\hat{} = \chi_{I^d}\hat{g}$. The Toeplitz operator on $PW(I^d)$ with symbol b is defined by

$$T_b(f) = P_{I^d}(bf) , \quad \text{for } f \in PW(I^d) . \tag{4}$$

And the Hankel operator on $PW(I^d)$ with symbol b is defined by

$$H_b(f) = P_{I^d}(b\bar{f}) , \quad \text{for } f \in PW(I^d) . \tag{5}$$