BETTER GOOD λ INEQUALITIES

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Introduction

In the early 1970s, D. Burkholder and R. Gundy introduced a technique for studying operators on L^p spaces. Their idea was to relate a pair of operators by a distribution function estimate which is now known as a "good-λ" inequality:

$$m(\{x \in \mathbb{R}^n: |Tf(x)| > 2\lambda, |Mf(x)| \leq \delta\lambda\})$$
$$\leq \epsilon\, m(\{x \in \mathbb{R}^n: |Tf(x)| > \lambda\}) .$$

Such an inequality implies that the L^p norm of Tf is bounded by the L^p norm of Mf. Thus, integrability results about M can be used to derive corresponding ones about T. Often, the method of proof allows one to replace Lebesgue measure by a weighted measure.

In many instances, this kind of result can be improved. Consider the situation when Tf is a maximal Calderón-Zygmund singular integral operator and Mf is the Hardy-Littlewood maximal function of f. R.R. Coifman and C. Fefferman proved [6]

$$w(\{x \in \mathbb{R}^n: Tf(x) > 2\lambda, Mf(x) \leq \delta\lambda\})$$
$$\leq \epsilon\, w(\{x \in \mathbb{R}^n: Tf(x) > \lambda\}) \qquad (0.1)$$

for any weight w in Muckenhoupt's A_∞ class. Our main result is an improved version of (0.1).

Theorem 1: Let $w \in A_\infty$ and $0 < \epsilon < 1$ There is a constant $C > 0$ such that