FOURIER TRANSFORM ASSOCIATED WITH HOLOMORPHIC DISCRETE SERIES

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1. INTRODUCTION

The Plancherel formula on semisimple Lie groups G implies that each L^2 function f on G has a decomposition: $f = f_P + {}^\circ f$, where f_P consists of wave packets and $^\circ f$ the discrete part of f, that is, a linear combination of the matrix coefficients of the discrete series of G. We assume that Ω=G/K, K is a maximal compact subgroup of G, is one of classical bounded symmetric domains. Then we shall give a characterization of $^\circ L^p(G)$ $(1 \leq p \leq 2)$, the discrete part of L^p functions on G, by using the Fourier transform associated with the holomorphic discrete series realized on a Bergman space on Ω. This characterization is related to the theory of the weighted Bergman spaces on Ω and the fractional derivatives of holomorphic functions on Ω.

In this introduction we shall treat the case of Ω= the open unit disk; in the rest of two sections we shall state the generalization on bounded symmetric domains of classical type.

First we shall recall the Fourier transform on the open unit disk D= $\{ z \in C ; |z| < 1 \}$. For $\lambda \in \mathbf{R}$ and $b \in \partial D$, the boundary of D, it is given by

$$\hat{f}(\lambda,b) = \int_D f(z)\left(\frac{1-|z|^2}{|z-b|^2}\right)^{\frac{1}{2}(-i\lambda+1)} dz.$$

As well known, we can identify D with the symmetric space G/K, where G= SU(1,1) and K=SO(2). By this identification G acts on D transitively and a function f(z) on D corresponds to the function $\tilde{f}(g)$ on G given by $\tilde{f}(g)$= $f(g\cdot 0)$, where $0 \in D$ and "$\cdot$" means the action of G. Then we can rewrite the above integral as follows.