DISINTEGRATION OF MEASURES ACCORDING TO THE DIMENSION AND ITS RELATION WITH RANDOM COVERINGS AND MULTIPLICATIVE CHAOS

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1. THE DISINTEGRATION THEOREM

Suppose we are given a locally compact metric or pseudometric space (T,dist) (here pseudometric means

$$\text{dist}(x,y) \le K(\text{dist}(x,z)+\text{dist}(z,y))$$

and metric is the case $K = 1$). A good example is $\mathbb{R}^d$ with the euclidean distance. We write $M^+(T)$ for the set of all positive and bounded Radon measures on T, $M^+_\alpha(T)$ for the subset of $M^+(T)$ which consists of α-Lipschitz measures, that is, measures σ which satisfy

$$\sigma(B) \le C_\sigma(\text{diam } B)^\alpha$$

for all balls B.

Given a Borel set B in T we consider the measures $\sigma \in M^+(T)$ carried by B (that is, $\sigma(B) = \sigma(T)$) and we write $\sigma \in M^+(B)$. We define $M^+_\alpha(B)$ in the same way. The capacitary (or Polya-Szegö) dimension of B is the supremum of the $\alpha>0$ such that $M^+_\alpha(B) \neq \{0\}$. If T is $\mathbb{R}^d$ with the usual metric the capacitarian dimension is the