OSCILLATORY INTEGRALS

Michael Cowling and *Shaun Disney*

We are interested in the behaviour at infinity of the Fourier transform $\hat{\mu}$ of the surface measure μ living on a smooth compact hypersurface S in $\mathbb{R}^{n+1}$, and in the transform $(f\mu)^{\wedge}$ of the product of certain functions f on S and μ. The aim of this work is to see how the decay of $\hat{\mu}$ and $(f\mu)^{\wedge}$ reflect the geometry of S. To simplify the statements of results, we assume that S is analytic.

The earliest and most important result on the decay of $(f\mu)^{\wedge}$ at infinity comes from the principle of stationary phase: in almost every direction σ in S^n,

$$(f\mu)^{\wedge}(\rho\sigma) = O(\rho^{-n/2}) \qquad \text{as } \rho \to +\infty.$$

More precisely, if σ is a generic direction, which means that the (finitely many) points $s_1, s_2, \ldots, s_K$ of S to which σ is normal are points of non-zero Gaussian curvature $\mathcal{K}$, then for smooth enough f (C^1 will do),

$$(1) \qquad (f\mu)^{\wedge}(\rho\sigma) = \Sigma_{k=1}^{K}\, c(k)\, f(s_k)\, |\mathcal{K}(s_k)|^{-1/2}\, e^{-i\rho\sigma . s_k}\, \rho^{-n/2} + o(\rho^{-n/2}) \qquad \text{as } \rho \to +\infty.$$

The constants $c(k)$ depend on the dimension n of S, and on whether σ is an inward or outward normal at s_k, relative to the principal curvatures. If σ is a non-generic direction, so that there is a point