JACKSON'S THEOREM FOR COMPACT CONNECTED LIE GROUPS

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This is an announcement of results which will appear in detail in the *J. Approx. Theory*.

Let E be a Banach space of periodic functions on $\mathbf{R}$, let $f \in E$ and let $n \geq 1$ be an integer. A basic problem in approximation theory is to estimate the quantity

$$\mathcal{E}_n(f) = \inf_t\{\|f - t\|_E\},$$

the infimum being taken over all trigonometric polynomials t of degree at most n. Jackson's Theorem is the fundamental "direct theorem" here; it asserts that if the r-th derivative $f^{(r)}$ exists in E (in the appropriate sense) and if E is suitable, then $\mathcal{E}_n(f) \leq C_r n^{-r}\omega_1(n^{-1}, f^{(r)}) = o(n^{-r})$ (see [6]). More precise versions of Jackson's Theorem provide estimates $\mathcal{E}_n(f) \leq C_r\omega_r(n^{-1}, f)$ for any $f \in E$, where $\omega_r(t, f)$ is the r-th modulus of continuity of f.

Jackson's Theorem extends in a straightforward way to periodic functions of k variables (i.e. functions on the group $\mathbf{T}^k$), and it is natural to ask whether it also applies to functions on nonabelian groups. We can prove that Jackson's Theorem is true for any compact connected Lie group:

THEOREM *Let $G \neq \{1\}$ be any compact connected Lie group. Let E denote one of the spaces $C(G)$ or $L^p(G)$, $1 \leq p < \infty$, and let $r \geq 1$ be an integer. Then there is a constant C_r and for each integer $n \geq 1$ there is a central trigonometric polynomial K_n of degree $\leq n$ such that*

$$\|f - K_n * f\|_E \leq C_r\omega_r(\frac{1}{n}, f)$$

for each $f \in E$.

Here a *central trigonometric polynomial of degree $\leq n$* is a linear combination of the characters χ_γ, where $\gamma \in \bar{K} \cap I^*$ and $||\gamma|| \leq n$ (The dual object $\hat{G}$ of G may be identified with a semilattice $\bar{K} \cap I^*$ as in [1, p. 242], and $||.||$ is a norm