INVARIANT DIFFERENTIAL OPERATORS ON SOME LIE GROUPS

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1. INTRODUCTION

A differential operator P on a Lie group G is said to be left (or right) invariant by G if it commutes with the action of G by left (or right) translations. We shall only consider linear differential operators. The algebra of left invariant linear differential operators on G is identified with the complexified universal enveloping algebra $U(\mathcal{G})$ of the Lie algebra $\mathcal{G}$ of G. Bi-invariant operators (i.e. operators which are both left and right invariant) then correspond to the elements of the centre $Z(\mathcal{G})$ of $U(\mathcal{G})$.

We study the problem of the existence of fundamental solutions for left invariant differential operators on Lie groups. We recall that if P is a differential operator on G, a fundamental solution for P is a distribution $E \in \mathcal{D}'(G)$ on G satisfying the equation $PE = \delta_G$, where δ_G is the Dirac distribution at the origin e_G of G.

Left invariant operators on a Lie group in general do not possess a global fundamental solution, but under additional conditions either on the operator or on the group, one can prove the existence of such solutions. For a review of the results we refer the reader for example to [1]. The present paper is a continuation of [1] and [2] and reports on some recent developments.