IDEAL STRUCTURE OF GROUPOID CROSSED PRODUCT C^* ALGEBRAS

Jean Renault and Bill Moran

We generalise to groupoid crossed products a theorem of E.Gootman and J. Rosenberg [GR], which asserts that every primitive ideal of the crossed product C^*–algebra is contained in an induced primitive ideal.

More precisely, if G is a second countable locally compact groupoid acting continually on a separable continuous field A of C^*–algebras over the unit space $G^{(0)}$ of G, then every representative L of the (full) crossed product C^*–algebra $C^*(G,A)$ weakly contains the representation induced from the restriction of L to the isotropy group bundle of the action of G on Prim A. The reverse inclusion holds if the action of G on Prim A is amenable.

Just as in [GR], the key ingredient of the proof is a "local cross–section theorem" which is better phrased in the following topological setting. If G is a topological groupoid, x a point of continuity of the isotropy and K a neighbourhood of x in G, which is symmetric and conditionally compact (c.c. for short)– that is, KL is compact for each compact subset L of $G^{(0)}$, then there exists a neighbourhood V of x in $G^{(0)}$ such that the relation $y \overset{K}{\sim} z$ if $y\,K\,z$ is non–void becomes on V an open and Hausdorff equivalence relation. This result is applied to the semi–direct product of the action of G on Prim A endowed with the regularized topology. Another tool is a G–equivariant version of a decomposition theorem for representations of C^*– algebras of E. Effros [E]. If L is a representation of $C^*(G,A)$, then there exist a transverse measure class Λ on Prim A and a covariant representation of (G,A) on a measurable field H of Hilbert spaces over Prim A, such that for almost every x the representation of A on H_x is homogeneous with kernel x, which provide by integration a representation unitarily equivalent to L.

This theorem, which compares a given representation with the representation induced from its restriction to the isotropy, does not give enough information on the