SPECTRAL SYNTHESIS OF ORBITS OF COMPACT GROUPS

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This talk is a contribution to the spectral synthesis in L^1 convolution algebras of noncommutative groups. Let us begin by recalling some ideas and results from the much better understood case of commutative groups. One way to consider spectral synthesis is as the attempt to classify the closed ideals in $L^1(G)$ for a locally compact abelian group G (or even in more general commutative Banach algebras).

With each closed ideal I in $L^1(G)$ there is associated a closed subset of the structure space $L^1(G)^\wedge \cong \hat{G}$, namely the hull $h(I) := \{\chi \in \hat{G} \mid \mathrm{Kern}_{L^1(G)}\, \chi \supseteq I\}$. This is clearly an invariant of the ideal. On the other hand, to each closed subset A of $\hat{G}$ one may form the kernel $k(A) := \bigcap_{\chi \in A} \mathrm{Kern}_{L^1(G)}\, \chi = \{f \in L^1(G)\, ;\ \hat{f} = 0 \text{ on } A\}$ where $\hat{f}$ denotes the Fourier transform of f. It is easy to see that $k(A)$ is the largest ideal I in $L^1(G)$ with $h(I) = A$. There is also a less obvious way to associate an ideal with A, namely

$$j(A) := \{f \in L^1(G) \mid \mathrm{supp}(\hat{f}) \text{ is a compact subset of } \hat{G} \setminus A\}^- .$$

It turns out that $j(A)$ is the smallest closed ideal I with $h(I) = A$. The classification problem reduces to: Describe (the ideal structure of) the algebra $k(A)/j(A)$ for closed subsets A of $\hat{G}$. The best possible situation is, of course, that $k(A)/j(A)$ is zero. In this case A is called a <u>set of synthesis</u> or a <u>Wiener set</u>. The next better situation is that