HANKEL OPERATORS ON THE PALEY-WIENER SPACE IN DISK

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1. INTRODUCTION

In [12], Rochberg has studied the Toeplitz and Hankel operators on the Paley-Wiener space in one dimension, and has got the characterizations for the Schatten-von Neumann class S_p criteria. In the end of [12], Rochberg proposed what are analogs of the results in several dimensions. In [11], Peng has studied the case of cube $I^d = \{\xi \in \mathbb{R}^d : -\pi < \xi_j < \pi \,, \, j = 1, \ldots, d\}$. In this paper, we study the case of disk $D = \{\xi \in \mathbb{R}^2 : |\xi| < 1\}$.

Let D denote the unit disk in $\mathbb{R}^2$, and let χ_D denote the characteristic function of D. The Paley-Wiener space on the unit disk, $PW(D)$, is defined to be the image of $L^2(D)$ under inverse Fourier transformations F^{-1}, i.e.

$$PW(D) = \left\{F^{-1}(\chi_D f) : f \in L^2(D)\right\} . \tag{1.1}$$

Let P_1, P_2 denote the projections defined by $(P_1 g)\hat{} = \chi_D \hat{g}$ and $(P_2 g)\hat{} = \chi_{2D} \hat{g}$, separately.

The Toeplitz operator on $PW(D)$ with symbol b is defined by

$$T_b(f) = P_1(bf) \,, \quad \text{for } f \in PW(D) . \tag{1.2}$$

And the Hankel operator on $PW(D)$ with symbol b is defined by

$$H_b(f) = P_1(b\overline{f}) \,, \quad \text{for } f \in PW(D) . \tag{1.3}$$

Because $PW(D)$ is preserved when taking complex conjugates, these two operators on $PW(D)$ are unitary equivalent. But as they have properties similar to those of classical Hankel operators (see below), we prefer the name Hankel operators in both cases.