PIECEWISE LINEAR FUNCTIONS AND SERIES EXPANSIONS IN TERMS OF DIRICHLET AND FEJÉR KERNELS

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If (x_n) is a sequence of vectors in a Banach space, there are various results which describe, in particular cases, when a subsequence (x_{n_k}) of (x_n) has certain properties, such as being basic, which are not possessed by the original sequence. For example, take the sequence $(e^{int})_{n=-\infty}^{\infty}$ in $C([0,2\pi])$. This sequence is not basic, but any lacunary subsequence of it is basic. Results along similar lines, for different sequences, may be found in [1], [2] and [3].

In this note are announced some results for certain sequences of vectors in $L^p(\mathbb{R})$ and $l^p(\mathbb{Z})$, for $1 \le p < \infty$, these vectors being linear on certain subintervals of $\mathbb{R}$ or $\mathbb{Z}$. This enables a certain characterization to be given of those functions which can be expanded in terms of a lacunary sequence of Dirichlet and Fejér kernels in $L^2(-\pi,\pi)$.

Let $1 \le p \le \infty$. Let $\alpha(0) = 0$ and let $(\alpha(n))$ be a given strictly increasing sequence of positive real numbers. The sequence $(\alpha(n))$ is said to be *lacunary* if there is a $\delta > 1$ such that $\alpha(n+1)\alpha(n)^{-1} \ge \delta > 1$, for all $n \in \mathbb{N}$. A Banach subspace $PL(p,\alpha)$ of $L^p(\mathbb{R})$ is defined as follows: $f \in PL(p,\alpha)$ if and only if $f \in L^p(\mathbb{R})$, f is even, f is zero on $\cap\{t : |t| \ge \alpha(n)\}$ and f is the restriction of a polynomial function of degree at most one upon each interval of the form $[\alpha(n-1),\alpha(n))$, for $n \in \mathbb{N}$. Let $PLC(p,\alpha)$ denote those functions in $PL(p,\alpha)$ which are continuous and let $PC(p,\alpha)$ denote those functions in $PL(p,\alpha)$ which are constant upon each interval of the form $[\alpha(n-1),\alpha(n))$, for $n \in \mathbb{N}$. If $\lim_{n\to\infty} \alpha(n) = \infty$, it is clear that $PLC(p,\alpha) \cap PC(p,\alpha) = \{0\}$. Let (u_n) be the sequence in $PL(p,\alpha)$ given by $u_{2n-1}(t) = 1$ for $|t| \le \alpha(n)$, $u_{2n-1}(t) = 0$ for $|t| > \alpha(n)$, and $u_{2n}(t) =$ maximum $(0, \alpha(n) - |t|)$.

THEOREM 1. *Let* $1 \le p < \infty$. *Then the following conditions are equivalent.*

(i) $(\alpha(n))$ *is lacunary,*

(ii) $PL(p,\alpha)$ *is the direct sum of* $PLC(p,\alpha)$ *and* $PC(p,\alpha)$,