On the transference theorem of Coifman and Weiss

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Suppose G is a unimodular locally compact group with a compact subgroup K and a polar decomposition with respect to K and a closed unimodular subgroup H. That is, we are assuming that the map $K \times H \times K \to G$, with $(k_1, h, k_2) \mapsto k_1 h k_2$, is surjective and there is a measurable function ω on H such that

$$\int_G f\, dm_G = \int_H \int_K \int_K f(k_1 h k_2)\omega(h) dm_K(k_1) dm_K(k_2) dm_H(h)$$

for all $f \in C_c(G)$. From Fubini's theorem we see that if f is a bi$-K-$invariant function on G then $\omega f|_H$ is an integrable function on H. We let $Cv_p(G)$ denote the Banach space of all bounded linear operators on the Lebesgue space $L^p(G)$ which commute with right translation by elements of G and we denote by $\|T\|_{Cv_p(G)}$ the norm of such an operator T. An integrable function f on G gives rise to a bounded, right-translation invariant operator $\lambda_G(f)\varphi := f * \varphi$ on each of the Lebesgue spaces $L^p(G)$. The norm of $\lambda_G(f)$ is determined by testing f against elements of the Herz-Figà-Talamanca algebra $A_p(G)$,

$$\| \lambda_G(f) \|_{Cv_p(G)} = \sup \left\{ \left| \int_G f g\, dm_G \right| \ : \ g \in A_p(G) \text{ and } \|g\|_{A_p(G)} \leq 1 \right\}.$$

The space $A_p(G)$ is defined in the following manner. Fix $1 < p < \infty$ and consider the projective tensor product $L^p(G) \widehat{\otimes} L^{p'}(G)$, where $1/p + 1/p' = 1$. There is a bounded linear map

$$P : L^p(G) \widehat{\otimes} L^{p'}(G) \to C_0(G),$$

given by $P(f \otimes g) = g * f^\vee$. The image of P is called $A_p(G)$ and is equipped with the quotient norm. That is, a function $\varphi \in A_p(G)$ has a series expansion

$$\varphi = \sum_{j=0}^{\infty} g_j * f_j^\vee$$

with $\sum_j \|g_j\|_{p'} \|f\|_p < \infty$, and the norm of φ is the infimum of all these sums. Every bounded linear operator $T : L^p(G) \to L^p(G)$ can be considered to be a bounded