A REMARK ON THE RELATIVE ENTROPY

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INTRODUCTION

The present article is a report of our joint works [4] and [5] with Mr. H. Yoshida on Pimsner-Popa's relative entropy $H(M|N)$ for a pair $M \supset N$ of finite von Neumann algebras. The notion of the relative entropy appeared first in Connes-Stormer's work [1] as a technical tool for finite dimensional algebras. In [6], M. Pimsner and S. Popa extended this notion for finite von Neumann algebras and made clear the relationship between $H(M|N)$ and Jones index $[M:N]$ for a pair $M \supset N$ of finite factors [3].

The aim of this article is to give complete formulas on $H(M|M^{\alpha})$ for an arbitrary action α of a locally compact group G on a finite von Neumann algebra M, applying Pimsner-Popa's deep results and our complementary general results. When M is a factor of type II_1, $H(M|M^{\alpha})$ is computed by using some conjugacy invariants of actions which are defined in a modified way of Jones' one [2].

§1 SOME GENERAL RESULTS.

Before entering in description, we fix some notations used hereafter. For a von Neumann algebra M, M^{+} = {all positive elements of M} and $Z(M)$ = the center of M. For a set S, $|S|$ = the cardinal number of S.