IRREDUCIBLE REPRESENTATIONS THAT CANNOT BE SEPARATED FROM THE TRIVIAL REPRESENTATION

E. Kaniuth

Let G be a locally compact group and $\hat{G}$ the dual space of G, i.e. the set of equivalence classes of irreducible unitary representations of G equipped with the usual topology. In general, $\hat{G}$ is far away from being a Hausdorff space. In fact, a connected group G has a Hausdorff dual iff G is an extension of a compact group by a vector group [2], and if G is discrete, then $\hat{G}$ is Hausdorff iff the centre of G has finite index in G. Therefore, it is reasonable to study the set of all those $\pi \in \hat{G}$ that cannot be separated from the trivial 1-dimensional representation 1_G, the so-called cortex cor(G) of G.

Interest in this closed subset of the dual also arose from the fact that the topology in the neighbourhood of 1_G is related to the group structur of G and to the cohomology of G in unitary representation spaces. It is well known (see [1]) that G has the Kazhdan property (T), i.e. $\{1_G\}$ is open in $\hat{G}$, iff $H^1(G,\pi) = 0$ for every unitary representation π of G. The following remarkable result has independently been obtained by Vershik and Karpushev [8] and by Larsen [7]: If G is second countable and $\pi \in \hat{G}$, then $H^1(G,\pi) \neq 0$ implies $\pi \in \mathrm{cor}(G)$.

Clearly, for $n \geq 3$, $SL(n,\mathbb{C})$, $SL(n,\mathbb{R})$ and $SL(n,\mathbb{Z})$ have a trivial cortex since they are groups with property (T). The cortex of $SL(2,\mathbb{C})$ consists of 1_G and the principal series representation which is usually denoted by $\pi_{2,0}$. $\mathrm{cor}(SL(2,\mathbb{R}))$ contains, except 1_G, two discrete series representations. It turns out that for every connected semi-simple Lie group G, cor(G) is finite [3]. To show this one observes that, for