FREDHOLM MODULES ASSOCIATED TO BRUHAT-TITS BUILDINGS

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To the memory of our friend Hugh Morris, a young and talented Australian mathematician

1. INTRODUCTION

In his Opus Magnum [Co], Connes defines an even unbounded Fredholm module over a C*-algebra A as a pair ($\mathcal{H}$, D), where $\mathcal{H}$ is a $\mathbb{Z}/2$-graded Hilbert space carrying a *-representation of A of degree 0, and D is an unbounded self-adjoint operator on $\mathcal{H}$, of degree 1, such that:

i) $(1 + D^2)^{-1}$ is compact

ii) The subalgebra $\mathfrak{A} = \{a \in A : [D,a] \text{ is bounded}\}$ is norm-dense in A.

Following [Co2], we say that an unbounded Fredholm module ($\mathcal{H}$, D) is ϑ-summable if, for any $t > 0$, the operator e^{-tD^2} is trace-class.

This condition is rather natural if one remembers the heat equation proof of the index theorem: Connes simply requires the "heat kernel" to be trace-class. In the case of the Dirac operator D on a compact riemannian spin manifold M, one even has p-summability in the sense of [Co]: $(1 + D^2)^{-p/2}$ is trace-class for $p > \dim M$. In particular $\operatorname{Tr} e^{-tD^2} = O(t^{-p/2})$ for $t \to 0$. However, as shown in [Co2], this condition of p-summability is too restrictive, as being too related to finite dimension and commutativity.

If G is a locally compact group, we define an unbounded G-Fredholm module as a pair ($\mathcal{H}$, D), where $\mathcal{H}$ is now a $\mathbb{Z}/2$-graded Hilbert space carrying a unitary representation of G, and D is as above, with condition ii) replaced by: