NORMAL SUBRELATIONS OF ERGODIC EQUIVALENCE RELATIONS

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§0 INTRODUCTION

In this paper we introduce the notion of a "normal" subrelation of an ergodic equivalence relation, and study some of its consequences. Details will appear elsewhere [2].

Throughout, we deal with a countable non-singular equivalence relation S on a standard non-atomic probability space (X,B,μ), as in [1] or [4]. Thus $S \subseteq X \times X$ is a Borel set, for each $x \in X$, $S(x) = \{y \in X : (y,x) \in S\}$ is countable, and for each $E \in B$ with $\mu(E) = 0$, $\mu(S(E)) = 0$ where $S(E) = \cup\{S(x) : x \in E\}$. For our purposes, there is no loss of generality in assuming that μ is a probability measure, and that S is ergodic i.e. for $E \in B$, $\mu(S(E)) \in \{0,1\}$; we will hence forth make these assumptions. In addition, we assume for simplicity that the measure μ is finite and invariant, as in [1] i.e. that S is of type II_1.

In recent years, much has been learned about the structure of such relations, up to isomorphism; here S_1 on (X_1,B_1,μ_1) is isomorphic to S_2 on (X_2,B_2,μ_2) if there is a bimeasurable map $\phi : X_1' \to X_2'$ between conull subsets of X_1 and X_2 for which $\mu_2 \circ \phi$ has the same null sets as μ_1, and for which $(x,y) \in S_1$ if and only if $(\phi(x),\phi(y)) \in S_2$. The reader is referred to [1,3,7] for a glimpse of what is known. Our interest however is not so much in the relations themselves, but in the possible subrelations R of a given ergodic equivalence relation S