Harmonic Analysis and Exceptional Representations of Semisimple Groups

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Introduction.

The purpose of this paper is to extend the results announced in the paper of Gilbert et.al. [3]. The authors showed that the concepts and techniques of Euclidean H^p theory can be applied to give realizations of ladder representations of $SO(4,1)$. (cf. Dixmier [2]). They single out for study a first-order differential operator $\mathfrak{d}$, which has the same principal symbol as the Calderon-Zygmund higher gradients operator on $\mathbb{R}^4$. The operator $\mathfrak{d}$ acts on functions with values in the space of spherical harmonics, which transform on the left according to the spherical harmonic representation $(m,0)$ of $SO(4)$. The authors showed:

1) $\mathfrak{d}$ is an elliptic differential operator.
2) The kernel of $\mathfrak{d}$, decomposed under the right-action of $SO(4)$, has a lowest K-type $(m,0)$, and the remaining K types are of the form $(m+j,0)$, $j>0$.
3) There is an embedding of limits of complementary series into the kernel of $\mathfrak{d}$, showing $\ker\mathfrak{d}$ is non-trivial.
4) Under the right action of $SO(4,1)$, the kernel of $\mathfrak{d}$ is irreducible and unitarizable.

The authors of [3] defined $\ker\mathfrak{d}$ as the intersection of the kernels of two Schmid operators (cf. Schmid [7]), and all the results of that paper followed from known results for discrete series. The ellipticity of $\mathfrak{d}$ followed from known embeddings of Schmid kernels into twisted Dirac operators; K-type information could be obtained from the Blattner multiplicity formulæ of Hotta and Parthasarathy ([4]); embeddings followed from known embeddings of discrete series into non-unitary principal series given by Knapp and Wallach [6]. Finally, unitarizability followed