ON THE L^1 BEHAVIOR OF EIGENFUNCTION EXPANSIONS AND SINGULAR INTEGRAL OPERATORS

Michael Christ[1] *and Christopher D. Sogge*[2]

1. INTRODUCTION

Let M be a compact, smooth manifold, without boundary, of dimension $n \geq 2$. Suppose that D is a pseudodifferential operator of the class $S^m_{1,0}$ on M, self-adjoint with respect to some measure μ with a smooth, nonvanishing density in local coordinates. Suppose further that either D is an elliptic differential operator whose principal symbol is real and nonnegative, or that $m = 1$ and D is a pseudodifferential operator whose symbol $a(x, \xi)$ has the property that

$$\lim_{s \to \infty} a(x, s\xi)$$

exists and is real and positive for all $\xi \neq 0$. For any such operator, $L^2(M, \mu)$ admits an orthogonal decomposition

$$L^2 = \bigoplus_{j=0}^{\infty} E_j$$

where each E_j is a finite-dimensional eigenspace of D with eigenvalue λ_j. These eigenvalues are distinct and form a discrete sequence which tends to $+\infty$. Denote by π_j the orthogonal projection of L^2 onto E_j. Then

$$S^0_t f = \sum_{\lambda_j \leq t} \pi_j f \to f$$

in L^2 norm as $j \to \infty$, for all $f \in L^2$, and

$$\|f\|^2_{L^2} = \sum \|\pi_j f\|^2_{L^2} .$$

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