REPRESENTATIONS OF INFINITE DIMENSIONAL GROUPS AND APPLICATIONS

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This talk reviews some recent work on representations of infinite dimensional groups which I have done jointly with Simon Ruijsenaars, Angas Hurst, Jill Wright and Keith Hannabuss. The main references are [1-5,8]. The point of view adopted here as a result of this work is the following: if $\mathcal{G}$ is a group whose representations one is interested in, then inject $\mathcal{G}$ into Aut $\mathcal{A}$, where $\mathcal{A}$ is a C*-algebra whose representation theory is reasonably well understood. Given an irreducible or factorial representation of $\mathcal{A}$ then, if it is true that $g.\pi$ and π are equivalent for all g in $\mathcal{G}$, the Hilbert space of π carries a projective representation $g \to \rho(g)$ of $\mathcal{G}$ where $\rho(g)$ is a unitary for each g in $\mathcal{G}$ such that

$$(1) \qquad \rho(g_1)\rho(g_2) = \sigma(g_1,g_2)\rho(g_1g_2)$$

with $\sigma(g_1,g_2)$ a unitary in $\pi(\mathcal{A})'$. Now σ is a 2-cocycle which may in general be difficult to compute. However for the groups we consider here (loop or gauge groups or the diffeomorphism group of the circle) extra information enables this cohomological problem to be overcome. For these cases we choose $\mathcal{A}$ to be the C*-algebra of the canonical anticommutation relations (variously known as the fermion algebra or the infinite dimensional Clifford algebra) over the complex Hilbert space H where H is either $L^2(\mathbb{R},\mathbb{C}^N)$ or $L^2(S^1,\mathbb{C}^N)$. This algebra is generated by $\{a(f),a(g)^* \mid f,g \in H\}$ subject to the relations.

$$(2) \qquad a(f)a(g)^* + a(g)^*a(f) = \langle f,g\rangle I; \quad a(f)a(g) + a(g)a(f) = 0,$$