On some C*-dynamical systems

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The problems addressed in this talk were partly motivated by attempts to classify the *-derivations defined on a class of smooth elements for an automorphic action α of a locally compact group G on a C*-algebra A. It has been proved that such a derivation δ has the form $\delta = d\alpha(X) + \tilde{\delta}$, where X is an element of the Lie algebra g of G and $d\alpha$ is a bounded derivation, under various circumstances, for example the following four:

(i) G is compact and there exists a faithful covariant representation π of A with $\pi(A^{\alpha})' \cap \pi(A)'' = \mathbb{C}1$. (Bratteli - Goodman and Longo, see [1, Theorem 2.9.2] and [8, Corollary 4.3]).

(ii) G is abelian, there exists sufficiently many G - invariant pure states and $\Gamma(\alpha) = \hat{G}$ (Batty - Ikunishi - Kishimoto, see [1, Theorem 2.9.10 and Corollary 2.9.17]).

(iii) G is abelian or compact, A is simple separable and there exists a sequence τ_n of automorphisms of A such that $\tau_n \alpha_g = \alpha_g \tau_n$ for all $n \in \mathbb{N}$ and $g \in G$, and $\lim_{n=\infty} \| \tau_n(x)y - y\tau_n(x) \| = 0$ for all x, y$\in$ A. (Bratteli - Kishimoto, see [1, Theorem 2.9.31])

Actually, using (iii) one proves.