SOME REMARKS ON INVERSE AND EXTREMAL EIGENVALUE PROBLEMS

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This paper gives a broad outline of recent work on both theoretical and numerical aspects of inverse and extremal eigenvalue problems, especially the developments that have taken place during my 1986-87 visit at the Centre for Mathematical Analysis and Mathematical Sciences Research Institute of the Australian National University. I would like to start by thanking all my hosts at the A.N.U. for their warm hospitality, especially Dr. Mike Osborne.

Let $A(x)$ be a smooth real $n \times n$ matrix function of a parameter vector $x \in \mathbb{R}^m$, and let $\lambda_i(x)$, $i = 1,\ldots,n$, be the eigenvalues of $A(x)$. The inverse eigenvalue problem is :

IEP: Given $\lambda_i^* \in \mathbb{C}$, $i = 1,\ldots,n$, and a domain $D \subseteq \mathbb{R}^m$, find $x \in D$ such that the sets $\{\lambda_i(x),\ i = 1,\ldots,n\}$ and $\{\lambda_i^*,\ i = 1,\ldots,n\}$, are the same. (We have not written $\lambda_i(x) = \lambda_i^*$ to avoid difficulties with inconsistent orderings.)

The most common version of the extremal eigenvalue problem is :

EEP: Given a domain $D \subseteq \mathbb{R}^m$, find $x \in D$ so that the spectral radius

$$\rho(x) = \max_{1 \le i \le n} |\lambda_i(x)|$$

is minimized over D.

The relationship between IEP and EEP is thus similar to the usual relationship between solving nonlinear systems of equations and nonlinear optimization. In particular, IEP may have no solution, while EEP must have a solution if D is compact, since $\rho(x)$ is continuous. The feature of IEP and EEP

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