REGULARITY OF SPACELIKE HYPERSURFACES

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Spacelike hypersurfaces play an important structural role in spacetimes, whether for the initial value problem (Cauchy surfaces), the structure of singularities (constant mean curvature surfaces), moments of time symmetry (maximal surfaces), cosmic time functions (surfaces of simultaneity), or for analyses of "the topology of space" (as opposed to spacetime). It is generally tacitly assumed that the surfaces in question are fairly "nice": achronal and without self-intersection, for instance. Suppose, instead, that a "hypersurface" is given, not as a closed, embedded, achronal submanifold (the nicest situation) but as an immersed submanifold, i.e., as a differentiable map $f : M^{n-1} \longrightarrow V^n$ (where V is the spacetime, with metric g) whose differential kills no tangent vectors in M (but $f(p) = f(q)$ is possible); this will be a spacelike "hypersurface" so long as the induced metric on M (f^*g) is Riemannian. What can be said about this?

In general, such a map need not be very nice, even if V is Minkowski n-space, $\mathbb{L}^n$: The image of f could easily be (for, say, $n = 3$) a spacelike ribbon intersecting itself or imitating a helical parking ramp. This is only possible, however, when $f(M)$ has an "edge"; any of various edgelessness or completeness assumptions render such behaviour impossible. For what other ambient spacetimes, besides Minkowski space, is this true? That is the subject of this paper.

How should the edgelessness of $f : M \longrightarrow V$ be phrased? One possibility would be simply to insist that $f(M)$ be closed; this, however, is far from sufficient, since the edge from one part of the surface could be made to lie in another part of the surface, thus keeping the image of f closed. The strongest assumption to make is that f be a proper map, i.e., that for any compact subset $K \subset V$, $f^{-1}(K)$ must also be compact; besides