THE SHAPE OF INFINITY

Richard P.A.C. Newman

1. INTRODUCTION

The remarkable developments that have taken place over the last few years in the field of low-dimensional topology are well-known. In 1982 Michael Freedman proved a 5-dimensional proper h-cobordism theorem which led him to a classification of compact simply connected topological 4-manifolds, and thus to a proof of the 4-dimensional Poincaré conjecture. And more recently, Simon Donaldson proved that compact simply connected smooth 4-manifolds have intersection forms of a very restricted type. This led to the now famous result that $\mathbb{R}^4$ admits an exotic differentiable structure. Donaldson's work was based on gauge theoretical techniques deriving from elementary particle physics, which may well benefit in consequence. One may contrast this with the situation for contemporary classical general relativity which, although being primarily concerned with manifolds of dimensions 3 and 4, has yet to make any such contact with the subtleties of low-dimensional topology. My objective here therefore is to show, in the context of a fundamental class of space-times, how relativity can give rise to topological problems that are of both mathematical and physical interest.

Consider an isolated massive body with a history extending indefinitely to the past. Suppose the gravitational field is too weak to generate collapse or to give rise to orbiting null geodesics akin to those at $r = 3m$ in Schwarzschild space-time. To make the situation even simpler, assume that there is an $\mathbb{R}^3$ Cauchy surface. Then the underlying space-time manifold is diffeomorphic to (standard) $\mathbb{R}^4$ and one may reasonably assume that all endless null geodesics originate from a past null infinity $\mathcal{I}^-$ and terminate at a future null infinity $\mathcal{I}^+$. As the space-time evolves, $\mathcal{I}^+$ is exposed to data on an increasingly large region of the