FACTORIZATION IN GROUP ALGEBRAS

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The following describes a connection with some automatic continuity problems of the study of probability measures and random walks on groups. The connection is via some questions concerning factorization in certain ideals in group algebras.

For this, let G be a locally compact group and M(G) denote the algebra of all bounded Borel measures on G with convolution product and total variation norm. The closed ideal in M(G), consisting of those measures which are absolutely continuous with respect to Haar measure, will be identified with the (topological) group algebra $L^1(G)$ and the closed subalgebra of M(G), consisting of discrete measures, will be identified with the (discrete) group algebra $\ell^1(G)$. Further, the (algebraic) group algebra, $\mathbb{C}G$, will be identified with the subalgebra of $\ell^1(G)$ consisting of functions with finite support. Since $L^1(G)$ is an ideal in M(G), the convolution product defines a right module action on $L^1(G)$ by each of the subalgebras, $\mathbb{C}G$, $\ell^1(G)$ and $L^1(G)$. Three automatic continuity problems now arise, namely, whether module homomorphisms from $L^1(G)$ to an arbitrary right $\mathbb{C}G$-, $\ell^1(G)$- or $L^1(G)$-module X are continuous.

The $L^1(G)$-module problem was solved by B. E. Johnson, see [6].

THEOREM 1. *Let* T *be a module homomorphism from* $L^1(G)$ *to an arbitrary right* $L^1(G)$-*module* X. *Then* T *is continuous.*

Proof. The proof is included in order to motivate an approach to the $\ell^1(G)$-module problem later. Let $(F_n)_{n=1}^{\infty}$ be a sequence in $L^1(G)$ which converges to zero in norm. Then, by an extension of Cohen's factorization theorem given in [6] there are F and F_n', n = 1,2,3,... in $L^1(G)$ such that

(i) $F_n = F*F_n'$, n = 1,2,3,...

(ii) $\lim_{n\to\infty} \|F_n'\| = 0$.