DERIVATIONS OF CONVOLUTION ALGEBRAS

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1. INTRODUCTION

Let w be a radical, algebra weight on $\mathbb{R}^+ = [0, \infty)$. That is, w satisfies $w(x+y) \leq w(x)w(y)$, and $w(x)^{1/x} \to 0$ as $x \to +\infty$. We will also assume that w is continuous on $\mathbb{R}^+$, and that $w(0) = 1$. We are interested in derivations on the algebra $L^1(w)$ consisting of Lebesgue measurable functions on $\mathbb{R}^+$ which are integrable with respect to the weight w, and on related algebras. The algebra $M(w)$ of Radon measures on $\mathbb{R}^+$ which have finite total variation with respect to w will also play an important part. By identifying functions with absolutely continuous measures, $L^1(w)$ is a closed ideal in $M(w)$. It is also true that $M(w)$ is isometrically isomorphic with the multiplier algebra of $L^1(w)$, a measure μ corresponding to a multiplier T by $Tf = \mu * f$. Our interest in derivations is related to some questions about automorphisms. We will mention one such question, but this article will concentrate on derivations. Unless otherwise indicated, all integrals occurring here will be over the domain $\mathbb{R}^+$.

We begin by fixing some notation. We write X for the operation of multiplication by the coordinate function: thus, if f is a function, Xf is the function defined by $Xf(x) = x\,f(x)$, and if μ is a measure, $X\mu$ is the measure defined by $d(X\mu)(x) = x\,d\mu(x)$. Similarly, if z is a complex number, e^{zX} denotes the operation of multiplication by the function $x \mapsto e^{zx}$. If λ is a real number, it is easy to check that $e^{\lambda X}w$ is a radical, algebra weight. Let A_λ denote $L^1(e^{-\lambda X}w)$, and let M_λ denote $M(e^{-\lambda X}w)$. Note that the algebras A_λ (respectively, M_λ) are all isomorphic. In fact, $e^{(\lambda-\rho)X}$ defines an isomorphism from A_ρ onto A_λ (respectively, M_ρ onto M_λ). Also note that $A_\lambda \supseteq A_\rho$ if $\lambda \geq \rho$, and the inclusion map is a continuous embedding of A_ρ onto a dense subalgebra

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