RINGS OF QUOTIENTS OF ULTRAPRIME BANACH ALGEBRAS. WITH APPLICATIONS TO ELEMENTARY OPERATORS

Martin Mathieu

1. INTRODUCTION

From its beginning, the investigation of non-commutative prime rings has been connected with the concept of (right) rings of quotients. The former begun by McCoy in [22] and continued by Johnson [15] strongly influenced the development of the latter, whose early stages are related with the names of Johnson [16], Utumi [29], Findlay and Lambek [7], and others. Both ideas have found important applications in many areas of algebra, for instance in the theory of GPI-rings. A comprehensive account of the state of art of rings of quotients was given in Stenström's book [25], where the more sophisticated approach via Gabriel topologies is used.

In the setting of *commutative* Banach algebras, Suciu studied algebras of quotients which can be normed ([26], [27]). For non-commutative prime normed algebras A over $\mathbb{C}$ there are, however, obstructions to endow the appropriate 'algebra of quotients', viz. Utumi's maximal ring of quotients, $Q_a(A)$, with a norm. The center C of $Q_a(A)$, the so-called *extended centroid of* A, is a field containing the center of A and, endowed with an algebra norm, would therefore coincide with $\mathbb{C}$ by Mazur's theorem. As often in Banach algebra theory, the truly non-commutative algebras thus play a distinguished role among general Banach algebras.

In the present paper we will introduce a topological version of primeness in the following sense. We call a normed algebra A *ultraprime* if its ultrapower $\hat{A}_{\mathcal{U}}$ with respect to some countably incomplete ultrafilter $\mathcal{U}$ is a prime algebra. Together with the corresponding notion of an ultraprime ideal and an intrinsic characterization (Lemma 3.1), some basic properties and examples of ultraprime normed algebras will be presented in Section 3. In particular, we prove that the center of every ultraprime normed algebra is trivial and that completion yields an ultraprime Banach algebra (both assertions