SOME REMARKS ABOUT IDEAS AND RESULTS OF TOPOLOGICAL HOMOLOGY

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The area which recently began to be referred to as "topological homology" by Michai Putinar, [40], and some other mathematicians, was initiated in 1962 by Kamowitz [1]. It was he who introduced the first important concept in the area by carrying into Banach algebra the purely algebraic notion of Hochschild cohomology groups, [0].

Let A be a Banach algebra, X a Banach A-bimodule. We call an n-linear continuous operator $f : A \times \ldots \times A \to X$ an *n-cochain.* These cochains form a Banach space denoted by $C^n(A,X)$, $n > 0$; we take $C^0(A,X)$ as X. Now let us consider the so-called *standard cohomological complex*

$$0 \longrightarrow C^0(A,X) \xrightarrow{\delta^0} \ldots \longrightarrow C^n(A,X) \xrightarrow{\delta^n} C^{n+1}(A,X) \longrightarrow \ldots \qquad (\mathcal{C}(A,X))$$

where δ^n is given by

$$(\delta^n f)(a_1,\ldots,a_{n+1}) = a_1 f(a_2,\ldots,a_{n+1}) + \sum_{k=1}^{n} (-1)^k f(a_1,\ldots,a_k a_{k+1},\ldots,a_n) + (-1)^{n+1} f(a_1,\ldots,a_n) a_{n+1}$$

(it is indeed a complex since $\delta^{n+1}\delta^n = 0$; $n \geq 0$) .

DEFINITION (Kamowitz, [1] Guichardet, [2]). The n-th cohomology of $\mathcal{C}(A,X)$ is called the *n-dimensional cohomology group of* A *with coefficients in* X (and is denoted by $\mathcal{H}^n(A,X)$.

In the spirit of Hochschild, Kamowitz applied the groups $\mathcal{H}^2(A,X)$ to the investigation of questions concerning the Wedderburn structure of some extensions of Banach algebras. Let us call an extension