SEMIGROUPS AND THE STRUCTURE OF WEIGHTED CONVOLUTION ALGEBRAS

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1. INTRODUCTION

Weighted convolution algebras on the half line $\mathbb{R}^+ = [0,\infty)$ arise naturally as the domains of the operational calculus maps determined by semigroups of bounded operators on a Banach space. For a continuous semigroup in a Banach algebra, which exists in every Banach algebra with bounded approximate identity [15, Th. 3.1, pp.35–36], this operational calculus map becomes an algebra homomorphism [15, pp.38–40], so weighted convolution algebras also arise as domains of Banach algebra homomorphisms. In fact, one of the motivations for the study of weighted convolution algebras is to learn more about the operational calculus map (see for example [15, Prob. 3.8, p.40], [11], [12, Th. 5.1]) and to use this map and structural features of weighted convolution algebras to study semigroups of operators and Banach algebras with a bounded approximate identity (see for instance [5], [15]). In this paper we report on research, much of it done jointly with F. Ghahramani and J.P. McClure, which, in a sense, goes in the other direction. We study homomorphisms and other properties of weighted convolution algebras by using semigroups of convolution operators defined on them.

Suppose that w is a positive Borel function on $[0,\infty)$ and that both w and $1/w$ are locally essentially bounded. Then $L^1(w)$ is the Banach space of (equivalence classes of) locally integrable functions f on $[0,\infty)$ for which fw is in $L^1(\mathbb{R}^+)$, with the inherited norm $\|f\| = \int_0^\infty |f(t)|\, w(t)\, dt$. Similarly, $M(w)$ is the space of locally finite measures for which the norm $\|\mu\| = \int_0^\infty w(t)\, d|\mu|(t) < \infty$. As usual, we consider $L^1(w)$ as a subspace of $M(w)$ by identifying f in $L^1(w)$ with $f(t)\, dt$ in $M(w)$. We are interested in the case that $L^1(w)$ is an algebra under the convolution product