

LOCALLY INNER DERIVATIONS

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1. INTRODUCTION

Let A be an infinite-dimensional, unital complex Banach algebra. An outstanding conjecture on ‘super-amenability’ or ‘contractibility’ (see e.g. P.C. Curtis and R. J. Loy [2], A.J. Helemskii [4]) is equivalent to the existence, for every such algebra A , of a non-inner, continuous derivation from A into some Banach A -bimodule.

In this paper we give a method for the construction of certain derivations into various A -bimodules (both Banach and Fréchet) of A -valued functions. A natural name for the derivations to be constructed is *locally inner*. The aim is then to construct a derivation that is locally inner, but not inner. These problems lead, rather naturally, to certain questions concerning the homological nature of some particular modules of A -valued functions.

There is, of course, a well-developed mathematical language in which to discuss questions concerning relations between local and global properties, namely the theory of presheaves and sheaves. The questions raised in the present paper are closely linked with the first (Čech) cohomology groups with coefficients in certain sheaves. The paper can however be read without a knowledge of sheaf cohomology. It is, of course, possible that to answer some of the problems might require more of the technical machinery.

2. SHEAVES OF A -VALUED FUNCTIONS

We shall introduce, in a concrete way, some ideas about presheaves and sheaves. The more general ‘abstract’ notion of a presheaf is not, in fact, very different.

Let A be a given unital Banach algebra and let X be a topological space. For every open $U \subseteq X$, let $\mathcal{C}(U, A)$ be the algebra of all continuous A -valued functions on U . We also consider $\mathcal{C}(U, A)$ as an A -bimodule, the module actions being multiplication on the