REGULARITY AND NONREGULARITY FOR SOLUTIONS OF PRESCRIBED CURVATURE EQUATIONS

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Our aim here is to describe some recent results and conjectures concerning the equations of prescribed m-th mean curvature

(1) $$H_m[u] = g(x)$$

on domains $\Omega \subset \mathbb{R}^n$. Here $H_m[u]$ is defined by

(2) $$H_m[u] = S_m(\kappa_1,\dots,\kappa_n) = \sum_{1\le i_1<\dots<i_m\le n} \kappa_{i_1}\cdots\kappa_{i_m}$$

for any integer m such that $1 \le m \le n$, where $\kappa_1,\dots,\kappa_n$ are the principal curvatures, relative to the upward normal, of the graph of the function u. The left hand side of (1) is therefore a function of Du and D^2u, so (1) is a nonlinear second order equation. The cases $m = 1$, $m = 2$ and $m = n$ correspond to the mean, scalar and Gauss curvatures respectively. The mean and Gauss curvature equations have been studied intensively and are quite well understood. The remaining cases are not as well understood, but results for the extreme cases $m = 1$ and $m = n$ give some indication of what one may expect in the general case.

Except for the case $m = 1$, equation (1) is not elliptic on all functions $u \in C^2(\Omega)$. However, Caffarelli, Nirenberg and Spruck [3], [4] and Ivochkina [9] have shown that (1) is elliptic for functions $u \in C^2(\Omega)$ such that at each point of Ω the vector of principal curvatures of the graph of u belongs to the convex cone $\Gamma_m = \Gamma_m^n$ which is the component containing the positive cone $\Gamma_+ = \{\lambda \in \mathbb{R}^n : \lambda_i > 0 \ \forall i\}$ of the set in $\mathbb{R}^n$ on which S_m is positive. For such solutions to exist we must obviously assume that g is positive in Ω. We shall call such u *admissible.* If the vector of