INNER PRODUCT ALGEBRAS AND THE FUNCTION THEORY OF ASSOCIATED DIRAC OPERATORS

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INTRODUCTION: The aim of this paper is to introduce a countably infinite number of algebras associated to R^n, each of which contains a generalization of the Cauchy–Riemann equations, and Cauchy integral formula. The first of these algebras is the Clifford algebra, and the associated analysis is called Clifford analysis [2]. We demonstrate that a large number of results from Clifford analysis carry over to these other algebras, including the formulae for Cauchy–Kowalewski extensions described in [9]. We utilise these formulae to describe Cauchy Kowalewski extensions of the kernel for the Fourier transform. Motivated by [4,8] this leads us to construct mutually annihilating idempotents in these algebras, and to associate new differential operators to this kernel. These idempotents enable us to construct from L^1 functions on R^{n-1} solutions of these differential operators in the upper and lower half spaces. We show that from these solutions we can construct solutions to other differential equations including the heat equation.

Inner Product Algebras: From R^n equipped with the inner product $<,>$ we can construct the Clifford algebra $A_n(1)$. By taking the orthonormal basis $e_1,\dots e_n$ of R^n we can construct the basis $1, e_1,\dots e_n,\dots,e_{j_1}\dots e_{j_r},\dots e_1\dots e_n$ of A_n, where $1 \le r \le n$ and $j_1 < \dots < j_r$. Moreover, $e_ie_j + e_je_i = -2\delta_{ij}$. One important property of $A_n(1)$ is that each non–zero vector $x \varepsilon R^n$ has a multiplicative inverse $x^{-1} = \frac{-x}{\|x\|^2}$.

The vector x^{-1} is the Kelvin inverse of the vector x. One way, [1], to construct $A_n(1)$ is to take the tensor algebra, $T(R^n)$, of R^n, ie the algebra