BOOLEAN ALGEBRAS OF PROJECTIONS OF UNIFORM MULTIPLICITY ONE

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A classical result of J. von Neumann states that if T is a bounded selfadjoint operator in a separable Hilbert space, then the following four algebras are the same: (i) the algebra of all bounded Borel functions of T; (ii) the weakly closed operator algebra generated by T; (iii) the uniformly closed operator algebra generated by the projections in the resolution of the identity for T; (iv) the bicommutant of T.

In Banach spaces (or, more generally, locally convex Hausdorff spaces) the analogues of selfadjoint operators are scalar-type spectral operators with real spectrum. The extent to which von Neumann's bicommutant theorem carries over to such operators T in locally convex spaces X is discussed in the survey article [6]. If X is a Banach space, then the algebras (i) - (iii) always coincide [6; Theorem 1]. The question of whether they also agree with (iv) is still not satisfactorily resolved in general. For instance, it is known that if the resolution of the identity for T has uniform multiplicity one, then the algebras (i) - (iii) do coincide with (iv), [6; Theorem 1]. Unfortunately, this condition is not necessary [6; Remarks 6 & 7]. For non-normable spaces X, even the equality of (i) - (iii), when appropriately formulated, is no longer valid in general. However, if the weakly closed operator algebra generated by the resolution of the identity of T is algebraically isomorphic to $C(\Lambda)$, for some completely regular Hausdorff space Λ, then equality does hold, [6; Theorem 2]. Under this restriction, it again turns out that uniform multiplicity one is a sufficient condition for the equality of (i) - (iv).

In practice it may be difficult to determine whether a given Boolean algebra (briefly, B.a.) of projections has uniform multiplicity one. However, it is often easier to establish whether or not a cyclic vector exists. Since the existence of a cyclic vector implies uniform multiplicity one, this gives at least some hope of testing for uniform multiplicity one in specific examples. Unfortunately, it is easy to produce examples of Boolean algebras of

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