Functional Calculus for Non-Commuting Operators

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1. Introduction

Let $\mathscr{B}$ be a unital Banach algebra and $a = (a_1, \ldots, a_m) \in \mathscr{B}^m$. We construct a functional calculus $\Phi_a : \mathscr{F} \to \mathscr{B}$ with a joint spectrum $\gamma(a)$. The space $\mathscr{F}$ is a Banach algebra of functions $f : \mathbb{R}^m \to \mathbb{C}$ and Φ_a is a bounded linear transformation with compact support supp (Φ_a) in $\mathbb{R}^m$. If the a_j commute then Φ_a is a homomorphism and if also f is a polynomial in a neighbourhood of supp (Φ_a) then $\Phi_a(f) = f(a)$. In the non-commuting case weaker properties are retained.

Our primary interest is in the case $\mathscr{B} = \mathscr{B}(X)$, the space of bounded linear operators on the Banach space X. However, it is convenient to formulate the results in the more general setting. This work extends that of Taylor [9], Anderson [1], McIntosh and Pryde [5] and Pryde [6].

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