

THE NATURAL GENERALIZATION OF THE NATURAL CONDITIONS OF
LADYZHENSKAYA AND URAL'TSEVA

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In the 1960's, Ladyzhenskaya and Ural'tseva [4] introduced a set of hypotheses for divergence structure elliptic operators which arise in a natural way and which lead to important estimates on the solutions of equations involving such operators. Here we discuss a more general set of hypotheses which achieve the same ends while also being the most general such, in a sense explained below.

Specifically divergence structure elliptic operators are those of the form

$$Qu = \operatorname{div} A(x, u, Du) + B(x, u, Du),$$

for some vector function A and scalar function B . Ladyzhenskaya and Ural'tseva assumed that A and B satisfied the hypotheses

$$(H1) \quad p \cdot A(x, z, p) \geq |p|^m - a_0 |z|^m - a_1,$$

$$(H2) \quad |A(x, z, p)| \leq a_2 |p|^{m-1} + a_3 |z|^{m-1} + a_4,$$

$$(H3) \quad |B(x, z, p)| \leq b_0 |p|^m + b_1 |p|^{m-1} + b_2 |z|^{m-1} + b_3$$