FOUR REMARKS CONCERNING THE FEYNMAN INTEGRAL

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REMARK ONE

The efforts of mathematicians in the area of functional integration are, from a certain point of view, focused too narrowly. That is, more specifically, the idea of a Feynman integral is not conceived in sufficient generality. This statement may seem ridiculous now when the Feynman integral attracts more attention than ever before, is generalized in various directions, and is used in contexts far beyond Feynman's original intention. (Instead of elaborating I would like to refer to the collection [3].) Therefore, to indicate what I have in mind I wish to invoke a historical analogy and to suggest an example.

As for the historical analogy, it may strike the listener (and the reader) as somewhat preposterous, but I do not have a better one: I wish to advert to the beginnings of the Integral Calculus. Some people, with a certain amount of justification, take for the origin of the Integral Calculus Archimedes' calculations of areas of some planar figures and volumes of some solid bodies. However, André Weil is right when he insists that crediting Archimedes with the invention of the Integral Calculus would be a historical nonsense. Indeed, we cannot yet speak of the Integral Calculus even some 2000 years later when Fibonacci calculated the area "under the curve $y = x^n$ in the interval $[0,1]$, for $n = 3,4,...,9$, and not even after Fermat calculated this area for arbitrary integral $n \geq 1$. To be sure, this is not to belittle the ingenuity of Archimedes or that of Fibonacci or Fermat. On the contrary, we cannot speak of Integral Calculus at those stages precisely because each of the mentioned calculations was based on a particular "trick" exploiting the specificity of the considered problem and requiring ingenuity far exceeding that which is now needed for the calculation of such sophisticated integrals as presented at the Tripos, say. What was still missing was an underlying principle or a general theory, and that emerged only in the works of Barrow, Leibniz and Newton. Only in the light of such a