

TOEPLITZ OPERATORS ON FOCK SPACES

Jan Janas

1. INTRODUCTION

This work gives a brief exposition of recent progress made in the theory of Toeplitz operators in Bargmann-Segal (Fock) space. Such operators have been studied in several papers of Berezin in the early seventies [3],[4] and also in [8]. Substantial advances in understanding their properties have been made in recent years due to the works of Berger and Coburn [5], [6]. Other related results are contained in [9], [10]. Despite the fact that there is a natural equivalence between Toeplitz operators in Fock space and pseudo-different operators in $L^2(\mathbb{R}^n)$, their study requires some specific methods. One of such methods, introduced in [3], is based on the idea of Berezin symbol of operators acting in the Fock space. This method has been successfully employed in [6] and [9]. Topics such as the theory of Toeplitz forms over Fock spaces developed in [12], and attempts to generalize the theory for Fock space over general Hilbert space [2], [11] are related, but we shall not discuss them here. This brief report almost certainly misses other works on Toeplitz operators of which we are not aware (done mainly by physicists).

The paper is divided into three parts. The first part introduces the Segal-Bargmann-Fock space $H^2(\mu)$ and its relation to $L^2(\mathbb{R}^n)$, Toeplitz operators in $H^2(\mu)$, and the Berezin symbol of operators acting in $H^2(\mu)$. The second part deals with bounded Toeplitz operators. The third part is devoted to unbounded Toeplitz operators in $H^2(\mu)$.

The material of this work is based mainly on the following papers [5], [6], [7], [8], [9], [10].

I would like to thank the organisers of the Conference "Operators in Analysis" held at Macquarie University, September 25-27, for allowing me to present this work at the Conference.