AN EXTENSION OF A THEOREM OF HALMOS TO ORLICZ SPACES, AND AN APPLICATION TO NEMITSKY OPERATORS

Grahame Hardy

1. INTRODUCTION

In our paper [3], we gave an extension to Orlicz-Sobolev spaces of a theorem of Marcus and Mizel (Theorem 2.1 of [5]) on the mapping of Sobolev spaces by Nemitsky Operators. However we did not give an extension of their theorem on demicontinuity (i.e., "strong → weak" continuity) of Nemitsky operators - Theorem 4.1 in [5]. Essentially this was because Marcus and Mizel, in their proof, used a theorem of Halmos (see [2]) on the mapping of Lebesgue spaces under composition by Borel measurable functions. We did not have a suitable (i.e., compatible with the hypotheses made in [3]) version available for Orlicz spaces. We present here the needed extension of Halmos' theorem to Orlicz spaces, and an extension, to Orlicz-Sobolev spaces, of Marcus and Mizel's theorem on demicontinuity.

2. DEFINITIONS

Throughout this section, Ω denotes a bounded domain in $\mathbb{R}^n$.

(i) Orlicz Spaces

We shall only give the definitions needed to set the notation which occurs in the statements of our theorems. Further details and references (for (ii) below also) can be found in [1].

An *N-function* is a real valued continuous convex even function on $\mathbb{R}$, such that $M(u)/u \to 0(\infty)$ as $u \to 0(\infty)$. The *complementary* N-function $\tilde{M}$ to M is defined by $\tilde{M}(v) = \sup_{u \geq 0} [u \mid v \mid - M(u)]$. We say that M satisfies the Δ_2-*condition* if there exists