SOLVABILITY OF DIFFERENTIAL OPERATORS ON SEMIRADIAL SEMIDIRECT PRODUCTS

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Let $G = N \rtimes K$ be the semidirect product of a simply connected nilpotent Lie group by a compact Lie group. Let $\underline{\underline{n}}_i$ be the derived series of $\underline{\underline{n}}$, defined by $n_0 = n$, $\underline{\underline{n}}_i = [\underline{\underline{n}}_{i-1}]$ for $i \geq 1$. We shall say that the semidirect product is **semiradial** if we can write $\underline{\underline{n}}_i = \underline{\underline{n}}^i + \underline{\underline{n}}_{i-1}$ as a K-space, in such a way that the K-invariants in $\underline{\underline{n}}^i$ form a commutative algebra.

Let P be a bi-K-invariant, left N invariant differential operator on G, and consider the **partial Fourier coefficients** of P, $(P_\wedge)_{\wedge \in \widehat{K}}$. These are K-invariant, $\mathcal{B}(\mathcal{H}_\wedge)$-valued differential operators on N defined for $\phi \in C^\infty(N)$ by

$$(P_\wedge \phi)(n) = P(\phi \otimes \wedge).$$

In [2], the following result was proved.

Theorem 1. *Let G and P be as above and suppose that Ω is a relatively compact set in G.*

Suppose further that for each integer a there is a constant C so that for all $\wedge \in \widehat{K}$

$$\| P_\wedge \|' \geq CN(\wedge)^{-a}. \qquad (**)$$

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