SURFACE MEASURES —
MAXIMAL FUNCTIONS AND FOURIER TRANSFORMS

Michael Cowling and Giancarlo Mauceri

Let S denote a smooth hypersurface in $\mathbf{R}^{n+1}$ with surface measure dS induced by the Lebesgue measure of $\mathbf{R}^{n+1}$. We fix a smooth nonnegative function w with compact support in $\mathbf{R}^{n+1}$ and consider the finite Borel measure μ with $d\mu = wdS$, which is carried by S. For any function f in the Schwartz space $\mathcal{S}(\mathbf{R}^{n+1})$ we denote by $M_t f$ the averages of f over the dilates of S —

$$M_t f(x) = \int_S f(x - ty)\, d\mu(y) \qquad \forall t \in \mathbf{R}^+, \quad \forall x \in \mathbf{R}^{n+1} \text{ —}$$

and by $M_* f$ the associated maximal function —

$$M_* f(x) = \sup_{t>0} |M_t f(x)| \qquad \forall x \in \mathbf{R}^{n+1}.$$

Our purpose is to determine the range of p's for which an ***a priori*** estimate of the form

$$\|M_* f\|_p \leq C \|f\|_p \qquad \forall f \in \mathcal{S}(\mathbf{R}^{n+1}),$$

holds; this estimate entails that the sublinear operator M_* extends to a bounded operator on the Lebesgue space $L^p(\mathbf{R}^{n+1})$, hereafter abbreviated to L^p. In the last decade, since Stein's work on the "spherical maximal function" [S1], [SW], this problem has attracted considerable attention [B], [CM1], [CM2], [G], [SS1], [SS2]. It turns out that, at least when $p < 2$, the range of p's for which the maximal operator M_* is bounded on L^p is determined by the decay at infinity of the Fourier transform $\hat{\mu}$ of the measure μ.