SCHAUDER ESTIMATES ON LIE GROUPS

Robert J. Burns and Derek W. Robinson

1. INTRODUCTION

Let $\partial_1, \ldots, \partial_d$ denote the partial differentiation operators on the usual L_p-spaces $L_p(\mathbb{R}^d; dx)$ and $\partial^\alpha = \partial_1^{\alpha_1} \ldots \partial_d^{\alpha_d}$ their products. Then the subspaces

$$L_{p;n} = \bigcap_{\alpha; |\alpha| \leq n} D(\partial^\alpha) ,$$

where $|\alpha| = \alpha_1 + \ldots + \alpha_d$, are Banach spaces with respect to the norms

$$\varphi \in L_{p;n} \mapsto \|\varphi\|_{p;n} = \sup_{\alpha; |\alpha| \leq n} \|\partial^\alpha \varphi\|_p .$$

Moreover, if $\Delta = -\sum_{i=1}^d \partial_i^2$ is the Laplacian then $L_{p;n} = D(\Delta^{n/2})$ and for each $p \in \langle 1, \infty \rangle$, each $n = 1, 2, \ldots$, and each $\lambda > 0$ there is a $C_{p,n,\lambda} > 0$ such that

$$C_{p,n,\lambda}^{-1} \|\varphi\|_{p;n} \leq \|(\lambda I + \Delta)^{n/2} \varphi\|_p \leq C_{p,n,\lambda} \|\varphi\|_{p;n} \tag{1.1}$$

for all $\varphi \in L_{p;n}$ (see, for example, [Tri] Section 2.2.3).

Thus the differential structure of the L_p-spaces coincides with the structure given by the Laplacian. Our primary aim is to prove similar properties for the differential structures associated with left, or right, translations on the L_p-spaces over a Lie group G.

It should be emphasized that the equivalence of the norms $\varphi \mapsto \|\varphi\|_{p;n}$ and $\varphi \mapsto \|(\lambda I + \Delta)^{n/2} \varphi\|_p$ is not valid if $p = 1$, or $p = \infty$. Nor does it hold on $C_0(\mathbb{R}^d)$. The domination properties

$$\|\varphi\|_{p;n} \leq C_{p,n,\lambda} \|(\lambda I + \Delta)^{n/2} \varphi\|_p ,$$