COMPENSATED COMPACTNESS AND THE BITING LEMMA

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In this talk I want to discuss some recent developments in the theory of compensated compactness, especially weak continuity of Jacobians of vector-valued functions and weak lower semicontinuity of variational integrals with quasiconvex integrands in the sense of Chacon's Biting Lemma. The characterization of weak continuous functionals has been an important tool for studying nonlinear partial differential equations and the calculus of variations. In particular, the "div-curl" lemma was instrumental in the work of Tartar [22] and DiPerna [14] on conservation laws, and the study of null Lagrangians was central to the work of Ball [4,5] on polyconvex functions and nonlinear elasticity.

The following result, known as Chacon's Biting Lemma, will be used to study weak continuity and weak lower semicontinuity of functionals. For proofs and more general statements of the lemma, see Brooks and Chacon [13], Balder [3], Slaby [24], Ball and Murat [10].

Theorem A. *(Biting Lemma) Let $\Omega \subset R^n$ be bounded and measurable and let $f^{(j)}$ be a bounded sequence in $L^1(\Omega)$. Then there exist a function $f \in L^1(\Omega)$, a subsequence $f^{(\nu)}$ of $f^{(j)}$ and a non-increasing sequence of measurable subsets $E_k \subset \Omega$ with*

$$\lim_{k\to\infty} \operatorname{meas}(E_k) = 0,$$

such that

$$f^{(\nu)} \rightharpoonup f \qquad \text{in } L^1(\Omega \setminus E_k)$$

as $\nu \to \infty$, for each fixed k.

The results E_k, which are removed ("bitten from") Ω, are associated with possible concentrations of the sequence $f^{(j)}$. Here and in the rest of the talk, $\rightharpoonup$ and $\overset{*}{\rightharpoonup}$ denote weak convergence and weak-$*$ convergence.

The following is the main weak continuity result of this talk.

Theorem 1. *Let $n \geq 2$, $\Omega \subset R^n$ be open and bounded and $u^{(j)} \rightharpoonup u$ in $W^{1,n}(\Omega, R^n)$. Then there exists a subsequence $u^{(\nu)}$ of $u^{(j)}$ such that*

$$\det Du^{(\nu)} \rightharpoonup \det Du \qquad \text{in the sense of the biting lemma in } \Omega.$$

Theorem 1 is almost optimal, since in general we can not expect that $\det Du_\nu \rightharpoonup \det Du$ in $L^1(\Omega)$ (see Ball and Murat [8, Counterexample 7.3]).