PLATEAU'S PROBLEM FOR MINIMAL SURFACES WITH A CATENOIDAL END

Friedrich Tomi*

In a preceding paper R. Ye and the author have shown that every rectifiable Jordan curve in $\mathbb{R}^3$ bounds a minimal immersion of the punctured disc which stretches out to infinity and has a flat end, i.e. outside some ball the surface is the graph of a bounded function defined on some exterior domain in the asymptotic tangent plane [TY]. In the present paper we are concerned with the corresponding problem for minimal surfaces with a catenoidal end, which means that at infinity the surface is required to be the graph of a logarithmically growing function and therefore ressembles the shape of a half catenoid.

We introduce the following notation: C denotes the standard catenoid in $\mathbb{R}^3$, i.e.

$$C = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | \sqrt{x_1^2 + x_2^2} = \cos h x_3\},$$

and T the closure of the non-simply connected component of $\mathbb{R}^3 \backslash C$; by $(\cos h)^{-1}$ we mean the positive branch of the inverse to $\cos h$, i.e. $(\cos h)^{-1}(\rho) = \ell n(\rho + \sqrt{\rho^2 - 1})$.

Setting $D_r := \{z \in \mathbb{C} || z| \leq r\}$ we state our result in the following.

Theorem. Let Γ be a rectifiable Jordan curve contained in T which generates $\pi_1(T)$. Then, for every $\lambda \in [-1, +1]$ there exists $\mu > 0$ and a conformal minimal immersion $u : D_1^0 \backslash \{0\} \to \mathbb{R}^3$ such that

(i) u extends continuously to $D_1 \backslash \{0\}$ and u maps ∂D_1 onto Γ topologically,

(ii) $|u(z)| \to +\infty (z \to 0)$,

(iii) for some $r > 0$, $u | D_r \backslash \{0\}$ is embedded and $u(D_r \backslash \{0\})$ is the graph of a function φ which is defined on an exterior domain in the (x_1, x_2)-plane and satisfies

$$\begin{aligned} &|\varphi(x_1, x_2)| \leq \mu \text{ if } \lambda = 0, \\ &|\varphi(x_1, x_2) - \lambda (\cos h)^{-1}(\rho / |\lambda|)| \leq \mu \text{ if } \lambda \neq 0, \end{aligned} \tag{1}$$

where $\rho = \sqrt{x_1^2 + x_2^2}$.

It would be interesting to study the corresponding existence problem in the class of graphs, i.e. to solve the exterior Dirichlet problem for the minimal surface equation. Some new results in this direction have been proved by Krust [K].

The proof of the above theorem is based on the same principle as in the flat case: the solution is obtained as the limit of a sequence of expanding minimal annuli spanned by

* Work done during a visit to the Center for Mathematical Analysis of the Australian National University at Canberra. This visit has been made possible by a grant of the Deutsche Forschungsgemeinschaft.