FURTHER EXISTENCE RESULTS FOR TWO POINT BOUNDARY VALUE PROBLEMS ARISING IN ELECTRODIFFUSION

H.B. Thompson

1. INTRODUCTION.

In [**3**] the author discusses a two–point boundary value problem which arises naturally in the study of biology as for example in the study of nerve conduction. The physical problem is basically the study of two ions with the same valency diffusing and migrating across a liquid junction such as a membrane. The junction separates two comparatively large electrically neutral reservoirs each containing electrolyte composed of ion species such as sodium and chloride. The reservoirs are stirred to maintain different but constant concentrations and the ions species have different diffusion constants. As the diffusion constant and the concentration gradient determine the rate of diffusion of a given ion species across the junction an electric field E results. This field varies in proportion to local concentration differences in the ion species. The electric field exerts a countervailing force on the ions. For large reservoirs, a steady state is reached in which macroscopically there is nett transfer of mass but not of charge and hence no electric current across the junction. Ion numbers are conserved. With two ion species this steady state model gives rise to a system of differential equations for the ionic concentrations and the electric field strength. Elimination of the ionic concentrations from the system leads to the following differential equation for the electric field:

$$\begin{aligned} y'' = y\{\lambda - (y(0)^2 - y^2)/2 + [l\lambda + (y(0)^2 - y(1)^2)/2]x\} - \\ [l\lambda + (y(0)^2 - y(1)^2)/2]D, \qquad x \in [0,1], \qquad (1) \end{aligned}$$

where after scaling the liquid junction occupies the region $0 \leq x \leq 1$. Here y is proportional to the electric field E and the parameters l, λ and D are functions of the physical constants such as the diffusion constant. Electrical neutrality in the reservoirs yields the boundary conditions:

$$y'(0) = 0 = y'(1). \qquad (2)$$

The parameter range of physical interest is $l, \lambda > 0$, and $-1 < D < 1$.

For detailed discussion of this model see Bass [**1,2**].