A Geometric Measure Theory Approach To Exterior Problems For Minimal Surfaces

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1 Introduction

The classical Plateau Problem in minimal surface theory is the following question:

Suppose $\Gamma \subset \mathbb{R}^3$ is a simple closed Jordan curve. Does there exist a surface $M \subset \mathbb{R}^3$ which has Γ as its boundary, $\partial M = \Gamma$, such that its area $|M|$ is as small as possible?

Using parametric methods the Plateau Problem was solved around 1930 independently by Douglas and Radó. Their solution was parametrized over a disk.

The same problem as well as its higher dimensional analogue was also investigated using Geometric Measure Theory (GMT). In the classical three dimensional case the GMT-approach gives the following beautiful result:

There exists an embedded smooth oriented submanifold $M \subset \mathbb{R}^3$ with boundary Γ, such that M has least area among all oriented surfaces S with $\partial S = \Gamma$.

Recently, starting with the work by Tomi and Ye [TY], corresponding exterior problems for minimal surfaces have attracted attention. In their paper Tomi and Ye concentrate on the parametric method. The reader is referred to Tomi's paper in this volume.

The GMT-approach to exterior problems I want to present here has its origins in the proof of the Positive Mass Conjecture in general relativity by