LOCAL TECHNIQUES FOR MEAN CURVATURE FLOW

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Consider immersions

$$x : M^n \to \mathbf{R}^{n+1}$$

of an n-dimensional manifold without boundary into Euclidean space. We say $M = x(M^n)$ moves by mean curvature if there exists a one-parameter family $x_t = x(\cdot, t)$ of immersions with corresponding images $M_t = x_t(M^n)$ satisfying

$$\begin{aligned} \frac{d}{dt}x(p,t) &= -H(p,t)\,\nu(p,t) \\ x(p,0) &= x_0(p) \end{aligned} \qquad p \in M^n \tag{1}$$

for some initial data x_0. Here $H(p,t)$ and $\nu(p,t)$ denote mean curvature and outer unit normal of the hypersurface M_t at $x(p,t)$. Using the well-known formula $\Delta x = -H\nu$ for hypersurfaces M in $\mathbf{R}^{n+1}$ we obtain the parabolic system of differential equations

$$\left(\frac{d}{dt} - \Delta\right) x = 0 \qquad \textit{on } M_t \tag{2}$$

where Δ denotes the Laplace-Beltrami operator on M_t.

Let us first mention a number of global results about mean curvature flow: In 1984, G.Huisken [11] proved that compact, convex initial surfaces converge asymptotically to round spheres. A corresponding result for convex curves in the plane was established by Gage and Hamilton [7]. Grayson [8] extended this to arbitrary embedded planar curves.