DISCRETE NORMS FOR THE CONVERGENCE OF BOUNDARY ELEMENT METHODS

G.A. CHANDLER

1. INTRODUCTION.

The simplest of elliptic problems is the interior Dirichlet problem for Laplace's equation. We are given a bounded region $\Omega \subset \mathbb{R}^2$ with a smooth boundary Γ, and want to find the unknown function U satisfying

$$-\Delta U(x) = 0\,, \quad x \in \Omega\,, \qquad \text{and}$$
$$U(x) = g(x)\,, \quad x \in \Gamma;$$

for some given function g.

As the fundamental solution for $-\Delta$ is

$$G(x,\xi) := \frac{1}{2\pi} \log \frac{1}{|x-\xi|}\,, \quad x,\xi \in \mathbb{R}^2\,,$$

we have Green's formula

$$\text{(1.1)} \qquad \int_\Gamma G(x,.)U_\nu - \int_\Gamma G(x,.)U = U(x)\,, \quad x \in \text{int}(\Omega)$$

$$\text{(1.2)} \qquad = \frac{1}{2}U(x)\,, \quad x \in \Gamma\,.$$

(For any $x \in \Gamma$, $\nu(x)$ is the outward normal at x, $U_\nu(x) = \nu(x).\nabla U(x)$, and $G_\nu(x,\xi) = \nu(\xi).\nabla_\xi G(x,\xi)$ is the normal derivative of G with respect to the second variable.) The

This work was supported by the Australian Research Council through the program grant *Numerical analysis for integrals, integral equations and boundary value problems.*