

## ON ENTIRE SOLUTIONS OF THE $p$ -LAPLACIAN

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### 1. Introduction.

The main purpose of this paper is to establish the existence of positive solutions of the equation

$$(1) \quad - \sum_{i=1}^n D_i (|\nabla u(x)|^{p-2} D_i u(x)) + c|u(x)|^{p-2} u(x) = \sum_{i=1}^N r_i(x) |u(x)|^{q_i-2} u(x),$$

in  $\mathbb{R}_n$ , where  $1 < p < \infty$  and  $c > 0$  is a constant,  $p < q_i < \frac{np}{n-p}$  if  $n > p$  and  $p < q_i < \infty$  if  $p \geq n$ . The functions  $r_i : \mathbb{R}_n \rightarrow [0, \infty)$  ( $i = 1, \dots, N$ ) satisfy the hypotheses  $(a_i)$  and  $(b_i)$  of Section 4, which depend on whether  $p < n$ ,  $p = n$  or  $p > n$ . We are also interested in the behaviour of solutions at infinity and in the question of the existence of multiple solutions.

The paper is organized as follows. In Sections 2 and 3 we describe an abstract setting for the equation (1). Namely, we consider the equation

$$(2) \quad Au = \nabla \Phi(u)$$

in a reflexive Banach space  $X$ , where  $A$  is a potential operator and  $\Phi$  is a real-valued functional on  $X$ . We follow here the ideas developed by Stuart in [18] and [20] for the equation

$$u = \nabla \Phi(u)$$