DEGREE THEORY

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(The first three are general references on degree theory)

LECTURE 1

The idea of degree theory is to give a "count" of the number of solutions of nonlinear equations but to count solutions in a special way so that the count is stable to changes in the equations. To see why the obvious count does not work well consider a family of maps $f_t(x)$ on R defined by $f_t(x) = x^2 - t$. As we vary t, f_t changes smoothly. For $t < 0$, it is easy to see that $f_t(x) = 0$ has no solution, $f_0(x) = 0$ has zero as its only solution while for $t > 0$, there are two solutions $\pm\sqrt{t}$. Hence the numbers of solutions changes as we vary t. Hence, to obtain something useful, we need a more careful count. A clue is that $\dfrac{\partial f_t}{\partial x}$ has different signs at the

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